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இலங்கைப் பரீட்சைத் திணைக்களம்
Department of Examinations, Sri Lanka

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2020
கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2020
General Certificate of Education (Adv. Level) Examination, 2020

සංයුක්ත ගණිතය I
இணைந்த கணிதம் I
Combined Mathematics I

10 E I

Part B

* Answer **five** questions only.

11.(a) Let $f(x)=x^2+px+c$ and $g(x)=2x^2+qx+c$, where $p, q\in\mathbb{R}$ and $c>0$. It is given that $f(x)=0$ and $g(x)=0$ have a common root α. Show that $\alpha=p-q$.

Find c in terms of p and q, and **deduce** that

(i) if $p>0$, then $p<q<2p$,

(ii) the discriminant of $f(x)=0$ is $(3p-2q)^2$.

Let β and γ be the other roots of $f(x)=0$ and $g(x)=0$ respectively. Show that $\beta=2\gamma$.

Also, show that the quadratic equation whose roots are β and γ is given by

$2x^2+3(2p-q)x+(2p-q)^2=0$

(b) Let $h(x)=x^3+ax^2+bx+c$, where $a, b, c\in\mathbb{R}$. It is given that x^2-1 is a factor of $h(x)$. Show that $b=-1$.

It is also given that the remainder when $h(x)$ is divided by x^2-2x is $5x+k$, where $k\in\mathbb{R}$. Find the value of k and show that $h(x)$ can be written in the form $(x-\lambda)^2(x-\mu)$, where $\lambda, \mu\in\mathbb{R}$.

12.(a) It is required to select a musical group **consisting** of **eleven** members from among five pianists, five guitarists, three female **singers** and **seven** male singers such that it includes **exactly** two pianists and **at least** four guitarists. Find the number of different such musical groups that can be selected.

Find also the number of musical groups among these, having exactly two female singers.

(b) Let $U_r=\dfrac{3r-2}{r(r+1)(r+2)}$ and $V_r=\dfrac{A}{r+1}-\dfrac{B}{r}$ for $r\in\mathbb{Z}^+$, where $A, B\in\mathbb{R}$.

Find the values of A and B such that $U_r=V_r-V_{r+1}$ for $r\in\mathbb{Z}^+$.

Hence, show that $\displaystyle\sum_{r=1}^{n}U_r=\frac{n^2}{(n+1)(n+2)}$ for $n\in\mathbb{Z}^+$.

Show that the infinite series $\displaystyle\sum_{r=1}^{\infty}U_r$ is **convergent and find** its sum.

Now, let $W_r=U_{r+1}-2U_r$ for $r\in\mathbb{Z}^+$. **Show that** $\displaystyle\sum_{r=1}^{n}W_r=U_{n+1}-U_1-\sum_{r=1}^{n}U_r$.

Deduce that the infinite series $\displaystyle\sum_{r=1}^{\infty}W_r$ is convergent and find its sum.

[see page eight

13. (a) Let $\mathbf{A} = \begin{pmatrix} a+1 & 0 \\ 1 & 1 \\ 0 & 1 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ a & 2 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} a & 1 \\ a & 2 \end{pmatrix}$, where $a \in \mathbb{R}$.

Show that $\mathbf{A}^{\mathrm{T}}\mathbf{B} - \mathbf{I} = \mathbf{C}$, where $\mathbf{I}$ is the identity matrix of order 2.

Show also that $\mathbf{C}^{-1}$ exists **if and only if** $a \neq 0$.

Now, let $a = 1$. Write down $\mathbf{C}^{-1}$.

Find the matrix $\mathbf{P}$ such that $\mathbf{CPC} = 2\mathbf{I} + \mathbf{C}$.

(b) Let $z, w \in \mathbb{C}$. Show that $|z|^2 = z\bar{z}$ and applying it to $z - w$,

show that $|z - w|^2 = |z|^2 - 2\,\mathrm{Re}\, z\bar{w} + |w|^2$.

Write a similar expression for $|1 - z\bar{w}|^2$ and show that $|z - w|^2 - |1 - z\bar{w}|^2 = -\left(1 - |z|^2\right)\left(1 - |w|^2\right)$.

Deduce that if $|w| = 1$ and $z \neq w$, then $\left|\dfrac{z - w}{1 - z\bar{w}}\right| = 1$.

(c) Express $1 + \sqrt{3}i$ in the form $r(\cos\theta + i\sin\theta)$, where $r > 0$ and $0 < \theta < \frac{\pi}{2}$.

It is given that $(1 + \sqrt{3}i)^m (1 - \sqrt{3}i)^n = 2^4$, where m and n are positive integers.

Applying De Moivre's theorem, obtain equations sufficient to determine the values of m and n.

14. (a) Let $f(x) = \dfrac{x(2x - 3)}{(x - 3)^2}$ for $x \neq 3$.

Show that $f'(x)$, the derivative of $f(x)$, is given by $f'(x) = \dfrac{9(1 - x)}{(x - 3)^3}$ for $x \neq 3$.

Hence, find the interval on which $f(x)$ is increasing and the intervals on which $f(x)$ is decreasing.

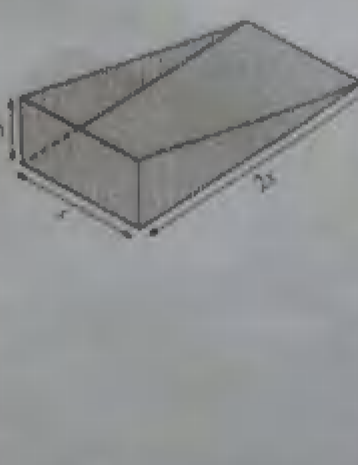
Also, find the coordinates of the turning point of $f(x)$.

It is given that $f''(x) = \dfrac{18x}{(x - 3)^4}$ for $x \neq 3$.

Find the coordinates of the point of inflection of the graph of $y = f(x)$.

Sketch the graph of $y = f(x)$ indicating the asymptotes, the turning point and the point of inflection.

(b) The adjoining figure shows the portion of a dust pan without its handle. Its dimensions in centimetres, are shown in the figure. It is given that its volume $x^2 h\ \mathrm{cm}^3$ is $4500\ \mathrm{cm}^3$. Its surface area $S\ \mathrm{cm}^2$ is given by $S = 2x^2 + 3xh$. Show that S is minimum when $x = 15$.



15.(a) It is given that there exist constants A and B such that

$$x^3 + 13x - 16 = A(x^2+9)(x+1) + B(x^2+9) + 2(x+1)^2 \text{ for all } x \in \mathbb{R}.$$

Find the values of A and B.

Hence, write down $\dfrac{x^3+13x-16}{(x+1)^2(x^2+9)}$ in partial fractions and

find $\displaystyle\int \frac{x^3+13x-16}{(x+1)^2(x^2+9)}\,dx$.

(b) Using integration by parts, evaluate $\displaystyle\int_0^1 e^x \sin^2 \pi x\,dx$.

(c) Using the formula $\displaystyle\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$, where a is a constant,

show that $\displaystyle\int_0^\pi x\cos^6 x\sin^3 x\,dx = \frac{\pi}{2}\int_0^\pi \cos^6 x\sin^3 x\,dx$.

Hence, show that $\displaystyle\int_0^\pi x\cos^6 x\sin^3 x\,dx = \frac{2\pi}{63}$.

16. Let $A \equiv (1, 2)$ and $B \equiv (3, 3)$.

Find the equation of the straight line l passing through the points A and B.

Find the equations of the straight lines l_1 and l_2 passing through A, each making an acute angle $\frac{\pi}{4}$ with l.

Show that the coordinates of any point on l can be written in the form $(1+2t, 2+t)$, where $t \in \mathbb{R}$.

Show also that the equation of the circle C_1 lying entirely in the first quadrant with radius $\frac{\sqrt{10}}{2}$, touching both l_1 and l_2, and its centre on l is $x^2 + y^2 - 6x - 6y + \frac{31}{2} = 0$.

Write down the equation of the circle C_2 whose ends of a diameter are A and B.

Determine whether the circles C_1 and C_2 intersect orthogonally.

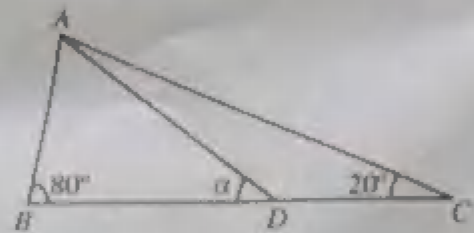
17.(a) Write down $\sin(A-B)$ in terms of $\sin A$, $\cos A$, $\sin B$ and $\cos B$.

Deduce that

(i) $\sin(90^\circ-\theta)=\cos\theta$, and

(ii) $2\sin 10^\circ=\cos 20^\circ-\sqrt{3}\sin 20^\circ$.

(b) In the usual notation, state the **Sine Rule** for a triangle ABC.



In the triangle ABC shown in the figure, $A\hat{B}C=80^\circ$ and $A\hat{C}B=20^\circ$. The point D lies on BC such that $AB=DC$. Let $A\hat{D}B=\alpha$.

Using the **Sine Rule** for suitable triangles, show that $\sin 80^\circ\sin(\alpha-20^\circ)=\sin 20^\circ\sin\alpha$.

Explain why $\sin 80^\circ=\cos 10^\circ$ and **hence**, show that $\tan\alpha=\dfrac{\sin 20^\circ}{\cos 20^\circ-2\sin 10^\circ}$.

Using **the** result in (a)(ii) above, **deduce** that $\alpha=30^\circ$.

(c) Solve the equation $\tan^{-1}(\cos^2 x)+\tan^{-1}(\sin x)=\dfrac{\pi}{4}$.

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